AD-A095 959 SOUTHERN METHODIST UNIV DALLAS TX DEPT OF MATHEMATICS F/G 12/1
NUMERICAL METHODS FOR BOUNDARY VALUE PROBLEMS.(U)
FEB 81 G W REDDIEN, E J DOEDEL DAAG29-80-C-0039

UNCLASSIFIED ARO-15673.4-M NL

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ARO 15673.4-M, and 17146.1-M

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LEVEL II

AD A095959

## TITLE

Numerical Methods For Boundary Value Problems.

## TYPE OF REPORT (TECHNICAL, FINAL, ETC.)

Final Report,

## AUTHOR (S)


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E. J. Doedel


## DATE

Feb[redacted]81

## U. S. ARMY RESEARCH OFFICE

Research Triangle Park, North Carolina

## CONTRACT/ GRANT NUMBER

DAAG 29-80-C-0039
DAAG29-79-

## INSTITUTION

Southern Methodist University



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81 3 2 007

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|---|
| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. AD-A095 959 | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) Numerical Methods for Boundary Value Problems | | 5. TYPE OF REPORT & PERIOD COVERED Final Report, June '78- Dec. '80 |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) G. W. Reddien, Southern Methodist U. E. J. Doedel , Concordia U., Montreal, | | 8. CONTRACT OR GRANT NUMBER(s) DAAG 29-80-C-0039 " 78 G 0126 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS G. W. Reddien, Mathematics Dept. Southern Methodist University Dallas, Texas 75275 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1L161102BH57-05 Math |
| 11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709 | | 12. REPORT DATE Feb. 17, 1981 |
| | | 13. NUMBER OF PAGES 5 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 15. SECURITY CLASS. (of this report) Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Collocation, bifurcation diagrams, stability of discretizations, two-point boundary value problems.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Collocation discretizations of two-point boundary value problems, including parameter dependent problems, are studied. Stability of certain discretizations is established, and their use for the computation of bifurcation diagrams, including the Hopf case, is investigated. Efficient reparameterizations for Hopf problems to allow computation around turning points are also given.

## Problem Statement

The properties of several discretizations of nonlinear boundary value problems in ordinary differential equations were studied. Special attention was given to the performance of these methods on problems arising in bifurcation theory and in optimal control. The methods considered were primarily collocation with piecewise polynomial functions.

## Summary of the Most Important Results

Methods introduced by Doedel (SINUM (1979)) were generalized and studied on general meshes. They were shown [1], [7], [11] to be unconditionally stable without a quasi-uniform assumption. Thus, these methods, which can be implemented very efficiently [1] could potentially be used in combination with a mesh selection strategy. This has not been investigated yet. The relation between the number of solutions of the discrete problem and the number of solutions of the continuous problem was also investigated. Examples were developed to show that these need not be the same in general, not even asymptotically as the mesh size goes to zero. Only under very restrictive assumptions on the differential equation can these multiplicities be guaranteed to agree for sufficiently small mesh sizes [7].

Collocation at Gauss points was shown to be a high order discretization of certain unconstrained optimal control problems. Best possible convergence rates and superconvergence results were established in [7]. The use of these methods with an adaptive mesh selection strategy was considered in [3]. These results are promising, but unfortunately incomplete.

Much of the effort under this grant was spent analyzing computational methods for parameter dependent two-point boundary value problems. The effects on the accuracy of collocation and certain finite difference discretizations in the neighborhood of bifurcation points were studied in [6], [8], [9] and [10]. Included here were both Hopf and simple bifurcation. The behavior of Newton's method as a computational scheme to solve the algebraic equations generated by the discretizations was studied in [4] and is now well understood. One important result obtained was the derivation and computational justification [9] of a reparameterization of periodic problems to allow for the computation of branches around turning points.

Consider the system $du/dt + SF(a,u) = 0$ where we seek $2\Pi$-periodic solutions. To find a neighboring solution $(S,a,u)$ a distance $r$ from any given solution, add the condition

$$\theta_1 (S-S_0)^2 + \theta_2 (a-a_0)^2 + \theta_3 ||u-v||^2 = r^2$$

where the $\theta_i$ are fixed weights and $||\cdot||$ is the $L^2$-norm. The solution is unique up to translation and can be anchored by requiring $(u(0)-v(0))$ to be orthogonal to $v'(0)$.

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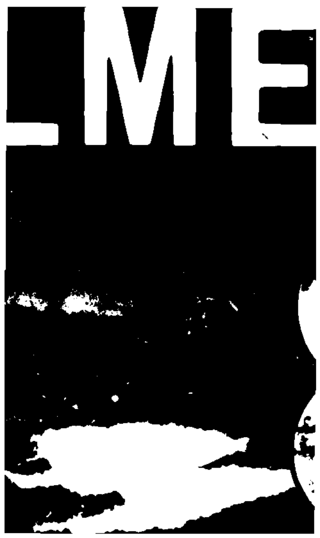
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